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THE HISTORY OF ANCIENT ASTRONOMY
PROBLEMS AND METHODS

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TABLE OF CONTENTS

Ce qui est admirable, ce n'est pas que le champ des étoiles soit si vaste, c'est que l'homme l'ait mesuré.—ANATOLE FRANCE, *Le Jardin d'Épicure.*

I. INTRODUCTION

1. In the following pages an attempt is made to offer a survey of the present state of the history of ancient astronomy by pointing out relationships with various other problems in the history of ancient civilization and particularly by enumerating problems for further research which merit our interest not only because they constitute gaps in our knowledge of ancient astronomy but because they must be clarified in order to lay a solid foundation for the understanding of later periods.

I wish to emphasize from the very beginning that the attitude taken here is of a very personal character. I do not believe that there is any single approach to the history of science which could not be replaced by very different methods of attack; only trivialities permit but one interpretation. I must confess still more: I cannot even pretend to be complete in the selection of topics essential for our understanding of ancient astronomy;[1] nor do I wish to conceal the fact that many of the steps which I myself have taken were dictated by mere accident. To mention only one example: without having been brought into contact with a recently purchased collection of Demotic papyri in Copenhagen, I would never have undertaken the investigation of certain periods of Hellenistic and Egyptian astronomy which now seem to me to constitute a very essential link between ancient and medieval astronomy. In other words, though I have always tried to subordinate any particular research problem to a wider program of systematic analysis, the impossibility of elaborate long-range planning has again and again been impressed upon me. The situation is comparable to entering a vast mountainous region on a single trail; one must simply follow the winding path, trying to give account of its general direction, but one can never predict with certainty what new vistas will be exposed at the next turn.

2. The enormous complexity of the study of ancient astronomy becomes evident if we try to make the first, and apparently simplest, step of classification: to distinguish between, say, Mesopotamian, Egyptian, and Greek astronomy, not to mention their direct successors, such as Hindu, Arabic, and medieval astronomy. Neither geographically nor chronologically nor according to language can clear distinctions be made. Entirely different conditions underlie the astronomy in Egypt of the Middle and New kingdoms than in the periods after the Persian conquest. Greek astronomy of Euclid's time has very little in common with Hipparchus' astronomy only a hundred and fifty years later. It is evident that it is of very little value to speak about a "Babylonian" astronomy regardless of period, origin, and scope. And, worst of all, the concept "astronomy" itself undergoes changes in meaning when we speak about different periods. The fanciful combination of a group of brilliant stars to form the picture of a "bull's leg" and the computation of the irregularities in the moon's movement in order to predict accurately the magnitude of an eclipse are usually covered by the same name! For methodological reasons it is obvious that a drastic restriction in terminology must be made. We shall here call "astronomy" only those parts of human interest in celestial phenomena which are amenable to mathematical

[1] Also the bibliography, given at the end, is very incomplete and is only intended to inform the reader where he can find further details of the specific viewpoint discussed here and to list the original sources.

treatment. Cosmogony, mythology, and applications to astrology must be distinguished as clearly separated problems —not in order to be disregarded but to make possible the study of the mutual influence of essentially different streams of development. On the other hand, it is necessary to co-ordinate intimately the study of ancient mathematics and astronomy because the progress of astronomy depends entirely on the mathematical tools available. This is in conformity with the concept of the ancients themselves: one need only refer to the original title of Ptolemy's "*Almagest*," namely, "Mathematical Composition."

3. The study of ancient astronomy will always have its center of gravity in the investigation of the Hellenistic-Roman period, represented by the names of Hipparchus and Ptolemy. From this center three main lines of research naturally emerge: the investigation of the previous achievements of the Near East; the investigation of pre-Arabic Hindu astronomy; and the study of the astronomy of late antiquity in its relation to Arabic and medieval astronomy. This last-mentioned extension of our program beyond antiquity proper is not only the natural continuation of the original problem but constitutes an integral part of the general approach outlined here. Astronomy is the only branch of the ancient sciences which survived almost intact after the collapse of the Roman Empire. Of course, the level of astronomical studies dropped within the boundaries of the remnants of the Roman Empire, but the tradition of astronomical theory and practice was never completely lost. On the contrary, the rather clumsy methods of Greek trigonometry were improved by Hindu and Arabic astronomers, new observations were constantly compared with Ptolemy's results, etc. This must be paralleled with the total loss of understanding of the higher branches of Greek mathematics before one realizes that astronomy is the most direct link connecting the modern sciences with the ancient. In fact, the work of Copernicus, Brahe, and Kepler can be understood only by constant reference to ancient methods and concepts, whereas, for example, the meaning of the Greek theory of irrational magnitudes or Archimedes' integrations were understood only after being independently rediscovered in modern times.

There are, of course, very good reasons for the fact that ancient astronomy extended with an unbroken tradition deep into modern times. The structure of our planetary system is such that it is simple enough to permit the achievement of relatively far-reaching results with relatively simple mathematical methods, but complicated enough to invite constant improvement of the theory. It was thus possible to continue successfully the "ancient" methods in astronomy at a time when Greek mathematics had long reached a dead end in the enormous complication of geometric representation of essentially algebraic problems. The creation of the modern methods of mathematics, on the other hand, is again most closely related to astronomy, which urgently required the development of more powerful new tools in order to exploit the vast possibilities which were opened by Newton's explanation of the movement of the celestial bodies by means of general principles of physics. The confidence of the great scientists of the modern era in the sufficiency of mathematics for the explanation of nature was largely based on the overwhelming successes of celestial mechanics. Essentially the same held for scholars in classical times. In antiquity, mathematical tools were not available to explain any

physical phenomena of higher complexity than the planetary movement. Astronomy thus became the only field of ancient science where indisputable certainty could be reached. This feeling of the superiority of mathematical astronomy is best expressed in the following sentences from the introduction to the *Almagest:* "While the two types of theory could better be called conjecture than certain knowledge —theology because of the total invisibility and remoteness of its object, physics because of the instability and uncertainty of matter— . . . . mathematics alone . . . . will offer reliable and certain knowledge because the proof follows the indisputable ways of arithmetic and geometry."[2]

II. EGYPT

4. A few words must be said about Egyptian mathematics before discussing the astronomical material. Our main source for Egyptian mathematics consists of two papyri[3]—certainly not too great an amount in view of the length of the period in question! Still, it seems to be a fair assumption that we are well enough informed about Egyptian mathematics. Not only are both papyri of very much the same type but all additional fragments which we possess match the same picture—a picture which is paralleled by economic documents in which occur precisely those problems and methods which we find in the mathematical papyri. The Egyptian mathematical texts, furthermore, find their direct continuation in Greek papyri,[4] which again show the same pattern. It is therefore safe to say that Egyptian mathematics never rose above a very primitive level. So far as astronomy is concerned, numerical methods are of primary importance, and, fortunately enough, this is the very part of Egyptian mathematics about which we are best informed. Egyptian arithmetic can be characterized as being predominantly of an "additive" character, that is, its main tendency is to reduce all operations to repeated additions. And, because the process of division is very poorly adaptable to such procedures, we can say that Egyptian mathematics does not provide the most essential tools for astronomical computation. It is therefore not surprising that none of our Egyptian astronomical documents requires anything more than simple operations with integers. Where the complexity of the phenomena exceeded the capacity of Egyptian mathematics, the strongest simplifications were adopted, consequently leading to little more than qualitative results.

5. The astronomical documents of purely Egyptian origin are the following: Astronomical representations and inscriptions on ceilings of the New Kingdom,[5] supplemented by the so-called "diagonal calendars" on coffin lids of the Middle Kingdom[6] and by the Demotic-Hieratic papyrus "Carlsberg 1."[7] Secondly, the Demotic papyrus "Carlsberg 9," which shows the method of determining new moons.[8] Though written in Roman

[2] *Almagest* I, 1 (ed. Heiberg I, 6, 11 ff.).

[3] Math. Pap. Rhind [Peet *RMP;* Chace *RMP*] and Moscow mathematical papyrus [Struve *MPM*]. For a discussion of Egyptian arithmetic see Neugebauer [1], for Egyptian geometry Neugebauer [2], and, in general, Neugebauer *Vorl.* The most recent attempt at a synthesis of Egyptian science, by Flinders Petrie (*Wisdom of the Egyptians* [London, 1940]), must unfortunately be considered as dilettantish not only because of its disregard of essential source material but also because of its lack of understanding for the mathematical and astronomical problems as such.

[4] The continuation of this tradition is illustrated by the following texts: Demotic: Revillout [1]; Coptic: Crum *CO*, No. 480, and Sethe *ZZ*, p. 71; Greek: Robbins [1] or Baillet [1]. For Greek computational methods in general, see Vogel [1].

[5] Examples: The Nut-pictures in the cenotaph of Seti I (Frankfort *CSA*) and Ramses IV (Brugsch *Thes.* 1) and analogous representations in the tombs of Ramses VI, VII, and IX.

[6] Cf. Pogo [1] to [4].

[7] Lange-Neugebauer [1].

[8] Neugebauer-Volten [1].

times (after A.D. 144), this text undoubtedly refers to much older periods and is uninfluenced by Hellenistic methods. A third group of documents, again written in Demotic, concerns the positions of the planets.[9] In this case, however, it seems to be very doubtful whether these tables are of Egyptian origin rather than products of the Hellenistic culture; we therefore postpone a discussion to the section on Hellenistic astronomy.[10] The last group of texts is again inscribed on ceilings and has been frequently discussed because of their representation of the zodiac.[11] There can be no doubt that these latter texts were deeply influenced by non-Egyptian concepts characteristic for the Hellenistic period. The same holds, of course, for the few Coptic astronomical documents we possess.[12] It is, finally, worth mentioning that not a single report of observations is preserved, in strong contrast to the abundance of observational records from Mesopotamia. It is hard to say whether this reflects a significant historical fact or merely that we are at the mercy of the accidents of excavation.

Speaking of negative evidence, three instances must be mentioned which play a more or less prominent role in literature on the subject and have contributed much to a rather distorted picture of Egyptian astronomy. The first point consists in the idea that the earliest Egyptian calendar, based on the heliacal rising of Sothis, reveals the existence of astronomical activity in the fourth millennium B.C. It can be shown, however, that this theory is based on tacit assumptions which are very implausible in themselves and that the whole Egyptian calendar does not presuppose any systematic astronomy whatsoever.[13] The second remark concerns the hypothesis of early Babylonian influence on Egyptian astronomical concepts.[14] This theory is based on a comparative method which assumes direct influence behind every parallelism or vague mythological analogy. Every concrete detail of Babylonian and Egyptian astronomy which I know contradicts this hypothesis. Nothing in the texts of the Middle and New Kingdom equals in level, general type, or detail the contemporaneous Mesopotamian texts. The main source of trouble is, as usual, the retrojection into earlier periods of a situation which undoubtedly prevailed during the latest phase of Egyptian history. This brings us to the third point to be mentioned here: the assumption of an original Egyptian astrology. First of all, there is no proof in general for the widely accepted assertion that astrology preceded astronomy. But especially in Egypt is there no trace of astrological ideas in the enormous mythological literature which we possess for all periods.[15]

[9] Neugebauer [3]. [10] Cf. below, p. 24.

[11] I know of the following representations of zodiacs: *No. 1* (Ptolemy III and V, i.e., 247/181 B.C.): northwest of Esna, North temple of Khnum (Porter-Moss *TB* VI, p. 118); *Nos. 2 and 3* (Ptolemaic or Roman): El-Salâmûni, Rock tombs (Porter-Moss *TB* V, p. 18); mentioned by L'Hôte, *LE*, pp. 86–87. *No. 4* (Ptolemaic-Roman; Tiberius): Akhmîm, Two destroyed temples (Porter-Moss *TB* V, p. 20); mentioned by Pococke *DE*, I, pp. 77–78. *No. 5* (Tiberius): Dendera, Temple of Ḥatḥor, Outer hypostyle (Porter-Moss *TB* VI, p. 49). *No. 6* (Augustus-Trajan): Dendera, Temple of Ḥatḥor, East Osiris-chapel central room, ceiling, west half (Porter-Moss *TB* VI, p. 99). *Nos. 7 and 8* (1st cent. A.D.): Athribis, Tomb (Porter-Moss *TB* V, p. 32). *No. 9* (Titus and Commodus): Esna, Temple of Khnum (Porter-Moss *TB* VI, p. 116). *No. 10* (Roman): Dealer in Cairo, publ. Daressy [1], pp. 126–27, and Boll, *Sphaera*, Pl. VI. Five other representations of the zodiacal signs are known from coffins, all from Ptolemaic or Roman times. On the other hand, the original Egyptian constellations are still found on coffins of the Saitic or early Ptolemaic periods.

[12] The only nonastrological Coptic documents known to me are the tables of shadow lengths published by U. Bouriant and Ventre-Bey [1].—P. Bouriant [1] did not recognize that the text published by him was a standard list of the planetary "houses" with no specific reference to Arabic astronomy.

[13] Neugebauer [4], Winlock [1], Neugebauer [5].

[14] Sponsored especially by the "Pan-Babylonian" school.

[15] It is interesting to observe how deeply imbedded is the assumption that astrology must precede as-

The earliest horoscope from Egyptian soil, written in Demotic, refers to A.D. 13;[16] the earliest Greek horoscope from Egypt concerns the year 4 B.C.[17] We shall presently see that the assumption of a very late introduction of astrological ideas into Egypt corresponds to various other facts.

6. It is much easier to show that certain familiar ideas about the origin of astronomy are historically untenable than to give an adequate survey of our real knowledge of Egyptian astronomy. A. Pogo is to be credited with the recognition of the astronomical importance of inscriptions on the lids of a group of coffins from the end of the Middle Kingdom,[18] apparently representing the setting and rising of constellations, though in an extremely schematic fashion. The constellations are known as the "decans" because of their correspondence to intervals of ten days. He furthermore saw the relationship between these simple pictures and the elaborate representations on the ceilings of the tombs belonging to kings of the New Kingdom.[19]

It can be safely assumed that the coffin lids are very abbreviated forms of contemporaneous representations on the ceilings of tombs and mortuary temples of the rulers of the Middle Kingdom. The logical place for these representations of the sky on ceilings explains their destruction easily enough. The earliest preserved ceiling, discovered in the unfinished tomb of Senmut, the vezir of Queen Hatshepsut,[20] is about three centuries later than the coffin lids. Then come the well-preserved ceiling in the subterranean cenotaph of Seti I[21] and its close parallels in the tomb of Ramses IV[22] and later rulers.[23] The difficulties we have to face in an attempt to explain these texts can best be illustrated by a brief discussion of the above-mentioned papyrus "Carlsberg 1." This papyrus was written more than a thousand years after the Seti text but was clearly intended to be a commentary to these inscriptions. In the papyrus we find the text from the cenotaph split into short sections, written in Hieratic, which are followed by a word-for-word translation into Demotic supplemented by comments in Demotic. The original text is frequently written in a cryptic form, to which the Demotic version gives the key. We now know, for instance, that various hieroglyphs were replaced by related forms in order to conceal the real contents from the uninitiated reader. How successfully this method worked is shown by the fact that one such sign, which is essential for the understanding of a long list of dates of risings and settings of the decans, was used at its face value for midnight instead of evening.[24] It is needless to emphasize what the recognition of such substitutions means for the correct understanding of astronomical texts. A complete revision of all previously published material is needed in the light of this new

tronomy. Brugsch called his edition of cosmogonic and mythological texts "astronomische und astrologische Inschriften" in spite of the fact that these texts do not betray the slightest hint of astrology.

16 Neugebauer [6].

17 *Pap. Oxyrh.* 804. From this time until A.D. 500 more than sixty individual horoscopes, fairly equally distributed in time, are known to me.

18 Cf. n. 6.

19 Some of Pogo's assumptions must, however, be abandoned, because they are based on the distinction of different types of such coffin inscriptions. A close examination of these texts (and also unpublished material) shows that all preserved samples belong to the same type. A systematic edition of all these texts is urgently needed if we are to obtain a solid basis for the study of Egyptian constellations.

20 Winlock [2], pp. 34 ff., reprinted in Winlock *EDEB*, pp. 138 ff., and Pogo [5]. The final publication has not yet appeared.

21 Frankfort *CSA*.

22 Brugsch *Thes.* I opposite pp. 174–75, but incomplete (cf. Lange-Neugebauer [1], p. 90).

23 Cf. n. 5.

24 Sethe, *ZAA*, p. 293, n. 1, and Lange-Neugebauer [1], p. 63.

insight into the Egyptian scheme of describing the rising and setting of stars the year round. One point, however, must be kept in mind in every investigation of Egyptian constellations. One must not ascribe to these documents a degree of precision which they were never intended to possess. I doubt, for example, very much whether one has a right to assume that the decans are constellations covering exactly ten degrees of a great circle on the celestial sphere. I think it is much more plausible that they are constellations spread over a more or less vaguely determined belt around the sky, just as we speak about the Milky Way. It is therefore methodically wrong to use these star lists and the accompanying schematic date lists for accurate computations, as has frequently been attempted.

The second Demotic astronomical document, papyrus Carlsberg 9, is much easier to understand and gives us full access to the Egyptian method of predicting the lunar phases with sufficient accuracy. The whole text is based on the fact that 25 Egyptian years cover the same time interval as 309 lunations. The 25 years equal 9125 days, which are periodically arranged into groups of lunar months of 29 and 30 days. The periodic repetition of this simple scheme corresponds, on the average, very well with the facts; more was apparently not required, and, we may add, more was not obtainable with the available simple mathematical means which are described at the beginning of this section. The purpose of the text was to locate the wandering lunar festivals within the schematic civil calendar, as is shown by a list of the "great" and "small" years of the cycle, which contain 13 or 12 lunar festivals, respectively.[25] Accordingly, calendaric problems are seen to be the activating forces here as well as in the decanal lists of the Middle and New Kingdom. The two Carlsberg papyri thus give us a very consistent picture of Egyptian stellar and lunar astronomy and its calendaric relations and are in best agreement with the level known from the mathematical papyri.

Before leaving the description of Egyptian science, brief mention should be made of the much-discussed question of the "scientific" character of Egyptian mathematics and astronomy. First of all, the word "scientific" must be clearly defined. The usual identification of this question with that of the practical or theoretical purpose of our documents is obviously unsatisfactory. One cannot call medicine or physics unscientific even if they serve eminently practical purposes. It is neither possible nor relevant to discover the moral motives of a scientist—they might be altruistic or selfish, directed by the desire for systematization or by interest in competitive success. It is therefore clear that the concept "scientific" must be described as a question of methods, not of motives. In the case of mathematics and astronomy, the situation is especially simple. The criterion for scientific mathematics must be the existence of the concept of proof; in astronomy, the elimination of all arguments which are not exclusively based on observations or on mathematical consequences of an initial hypothesis as to the fundamental character of the movements involved. Egyptian mathematics nowhere reaches the level of argument which is worthy of the name of proof, and even the much more highly developed Babylonian mathematics hardly ever displays a general technique for proving its procedures.[26]

[25] The "great" and "small" years (already mentioned in an inscription of the Middle Kingdom) have given rise to much discussion (cf., e.g., Ginzel *Chron.*, I, pp. 176–77) which can now be completely ignored.

[26] See the discussion in Neugebauer *Vorl.*, pp. 203 ff.

Egyptian astronomy was satisfied with a very rough qualitative description of the phenomena—here, too, we miss any trace of scientific method. The first scientific attack of mathematical problems was made in the fifth century B.C. in Greece. We shall see that scientific astronomy can be found shortly thereafter in Babylonian texts of the Seleucid period. In other words, the enormous interest of the study of pre-Hellenistic Oriental sciences lies in the fact that we are able to follow the development far back into *pre*-scientific periods which saw the slow preparation of material and problems which deeply influenced the shape of the real scientific methods which emerged to full power for the first time in the Hellenistic culture. It is a serious mistake to try to invest Egyptian mathematical or astronomical documents with the false glory of scientific achievements or to assume a still unknown science, secret or lost, not found in the extant texts.

III. MESOPOTAMIA

7. Turning to Babylonian astronomy, one's first impression is that of an enormous contrast to Egyptian astronomy. This contrast not only holds in regard to the large amount of material available from Mesopotamia but also with respect to the level finally reached. Texts from the last two or three centuries B.C. permit the computation of the lunar movement according to methods which certainly rank among the finest achievements of ancient science—comparable only to the works of Hipparchus and Ptolemy.

It is one of the most fascinating problems in the history of ancient astronomy to follow the different phases of this development which profoundly influenced all further events. Before giving a short sketch of this progress as we now restore it according to our present knowledge, we must underline the incompleteness of the present state of research, which is due to the fact that we do not yet have reliable and complete editions of the text material. The observation reports addressed to the Assyrian kings were collected by R. C. Thompson[27] and in the editions of Assyrian letters published and translated by Harper,[28] Waterman,[29] and Pfeiffer;[30] much related material is quoted in the publications of Kugler,[31] Weidner,[32] and others. But Thompson's edition gives the original texts only in printed type, subject to all the misunderstandings of this early period of Assyriology, and very little has been done to repair these original errors. Nothing short of a systematic "corpus" of all the relevant texts can provide us with the requisite security for systematic interpretation. The great collection of astrological texts, undertaken by Virolleaud[33] but never finished, confronts the reader with still greater difficulties, because Virolleaud composed complete versions from various fragments and duplicates without indicating the sources from which the different parts came. And, finally, the tablets dealing with the movement of the moon and the planets were discussed and explained in masterly fashion by Kugler;[34] but here, too, a systematic edition of the whole material is necessary.[35] Years of systematic work will be needed before the foundations for a reliable history of the development of Babylonian astronomy are laid.

8. Kugler uncovered step by step the ingenious methods by which the ephemer-

[27] Thompson *Rep.* (1900).

[28] Harper *Letters.*

[29] Waterman *RC.*

[30] Pfeiffer *SLA.*

[31] Kugler *SSB* and Kugler *MP.*

[32] Weidner *Hdb.*, Weidner [1], [2], and numerous articles in the pre-war volumes of *Babyloniaca.*

[33] Virolleaud *ACh.*

[34] Kugler *BMR* and *SSB.*

[35] Such an edition by the present author is in preparation; it is quoted in the following as *ACT.*

ids of the moon and the planets which we find inscribed on tablets ranging from 205 B.C. to 30 B.C. were computed.[36] It can justly be said that his discoveries rank among the most important contributions toward an understanding of ancient civilization. It is very much to be regretted that historians of science often quote Kugler but rarely read him;[37] by doing this, they have disregarded the newly gained insight into the origin of the basic methods in exact science. This is not the place to describe in detail the Babylonian "celestial mechanics," as it might properly be called; that will be one of the tasks of a history of ancient astronomy which remains to be written. A few words, however, must be said in order to render intelligible the relationship between Babylonian and Greek methods. The problem faced by ancient astronomers consisted in predicting the positions of the moon and the planets for an extended period of time and with an accuracy higher than that obtainable by isolated individual observations, which were affected by the gross errors of the instruments used. All these phenomena are of a periodic character, to be sure, but are subject to very complicated fluctuations. All that we know now seems to point to the following reconstruction of the history of late Babylonian astronomy. A systematic observational activity during the Late Assyrian and Persian periods (roughly, from 700 B.C. onward) led to two different results. First, the collected observations provided the astronomers with fairly accurate average values for the main periods of the phenomena in question; once such averages were obtained, improvements could be furnished by scattered observational records from preceding centuries. Secondly, from individual observations, for example, of the moment of full moon[38] or of heliacal settings, etc., short-range predictions could be made by methods which we would call linear extrapolation. Such methods are frequently sufficient to *exclude* certain phenomena (such as eclipses) in the near future and, under favorable conditions, even to predict the date of the next phenomenon in question. After such methods had been developed to a certain height, apparently one ingenious man conceived a new idea which rapidly led to a systematic method of long-range prediction. This idea is familiar to every modern scientist; it consists in considering a complicated periodic phenomenon as the result of a number of periodic effects, each of a character which is simpler than the actual phenomenon.[39] The whole method probably originated in the theory of the moon, where we find it at its highest perfection. The moments of new moons could easily be found if the sun and moon would each move with constant velocity. Let us assume this to be the case and use average values for this ideal movement; this gives us average positions for the new moons. The actual movement deviates from this average but oscillates around it periodically. These deviations were now treated

[36] The first tentative (but very successful) steps were made by Epping *AB* (1889). Then follow Kugler's monumental works *BMR* (1900) and *SSB* (published between 1907 and 1924), supplemented by Schaumberger's explanation of the determination of first and last visibility of the moon (1935) and continued by the present author with respect to the theory of latitude and eclipses (Neugebauer [8], [9], Pannekoek [2] and van der Waerden [1]). The theory of planets is treated in Kugler *SSB*, to be supplemented by Pannekoek [1], Schnabel [2], and van der Waerden [2]. All previously published texts and much unpublished material will be contained in Neugebauer *ACT*. The whole material amounts to about a hundred ephemerids for the moon and the planets, covering the abovementioned two centuries.

[37] Abel Rey, *La Science orientale avant les grecs* (Paris, 1930), and E. Zinner, *Geschichte der Sternkunde* (Berlin, 1931), are brilliant examples showing complete ignorance of Kugler's results.

[38] Frequently mentioned in the "reports" to the Assyrian court (e.g., Thompson *Rep.*).

[39] A classic example is the treatment of sounds as the result of the superposition of pure harmonic vibrations.

as new periodic phenomena and, for the sake of easier mathematical treatment, were considered as linearly increasing and decreasing. Additional deviations are caused by the inclination of the orbits. But here again a separate treatment, based on the same method, is possible. Thus, starting with average positions, the corrections required by the periodic deviations are applied and lead to a very close description of the actual facts. In other words, we have here, in the nucleus, the idea of "perturbations," which is so fundamental to all phases of the development of celestial mechanics, whence it spread into every branch of exact science.

We do not know when and by whom this idea was first employed. The consistency and uniformity of its application in the older of the two known "systems" of lunar texts point clearly to an invention by a single person. From the dates of the preserved texts, one might assume a date in the fourth or third century B.C.[40] This basic idea was applied not only to the theory of the moon (in two slightly modified forms) but also to the theory of the planets. In this latter theory the main point consists in refraining from an attempt to describe directly the very irregular movement, substituting instead the separate treatment of several individual phenomena, such as opposition, heliacal rising, etc.; each of these phenomena is treated with the methods familiar from the lunar theory as if it were the periodic movement of an independent celestial body. After dates and positions of each characteristic phenomenon are determined, the intermediate positions are found by interpolation between these fixed points.[41] It must be said, however, that the planetary theory was not developed to the same degree of refinement as the lunar theory; the reason might very well be that the lunar theory was of great practical importance for the question of the Babylonian calendar: whether a month would have 30 or 29 days. For the planets no similar reason for high accuracy seems to have existed, and it was apparently sufficient merely to compute the approximate dates of phenomena, which, in addition, are frequently very difficult to observe accurately.

We cannot emphasize too strongly that the essential point in the above-described methods lies not in the comparatively high accuracy of the results obtained but in their fundamentally new attitude toward the whole problem. Let us, as a typical example, consider the movement of the sun.[42] Certain simple observations, most likely of the unequal length of the seasons, had led to the discovery that the sun does not move with constant velocity in its orbit. The naïve method of taking this fact into account would be to compute the position of the sun by assuming a regularly varying velocity. It turned out, however, that considerable mathematical difficulties were met in computing the syzygies of the moon according to such an assumption. Consequently, another velocity distribution was substituted, and it was found that the following "model" was satisfactory: the sun moves with two different velocities over two unequal arcs of the ecliptic, where velocities and arcs were determined in such a fashion that the initial empirical facts were correctly explained and at the same time the computation of the conjunctions became suffi-

[40] The attempts to determine a more precise date (Schnabel *Ber.*, pp. 219 ff., and Schnabel [1], pp. 15 ff.) are based on unsatisfactory methods. The generally accepted statement that Naburimannu was the founder of the older system of the lunar theory relies on nothing more than the occurrence of this name in one of the latest tablets in a context which is not perfectly clear.

[41] This is shown by a tablet for Mercury, to be published in Neugebauer *ACT*. The interpolation is not simply linear but of a more complicated type known from analogous cases in the lunar theory.

[42] For details see Neugebauer [10] and [9] § 2.

ciently simple. It is self-evident that the man who devised this method did not think that the sun moved for about half a year with constant velocity and then, having reached a certain point in the ecliptic suddenly started to move with another, much higher velocity for the rest of the year. His problem was clearly this: to make a very complicated problem accessible to mathematical treatment with the only condition that the final consequences of the computations correctly correspond to the actual observations—in our example, the inequality of the seasons. The Greeks[43] called this a method "to preserve the phenomena"; it is the method of introducing mathematically useful steps which in themselves need not be of any physical significance. For the first time in history, mathematics became the leading principle for the structure of physical theories.

9. It will be clear from this discussion that the level reached by Babylonian mathematics was decisive for the development of such methods. The determination of characteristic constants (e.g., period, amplitude, and phase in periodic motions) not only requires highly developed methods of computation but inevitably leads to the problem of solving systems of equations corresponding to the outside conditions imposed upon the problem by the observational data. In other words, without a good stock of mathematical tools, devices of the type which we find everywhere in the Babylonian lunar and planetary theory could not be designed. Egyptian mathematics would have rendered hopeless any attempt to solve problems of the type needed constantly in Babylonian astronomy. It is therefore essential for our topic to give a brief sketch of Babylonian mathematics.

[43] E.g., Proclus, *Hypotyposis astron. pos.* v. 10 (ed. Manitius, 140, 21).

I think it can be justly said that we have a fairly good knowledge of the character of mathematical problems and methods in the Old Babylonian period (*ca.* 1700 B.C.). Almost a hundred tablets from this period are published;[44] they contain collections of problems or problems with complete solutions—amounting to far beyond a thousand problems. We know practically nothing about the Sumerian mathematics of the previous periods and very little of the interval between the Old Babylonian period and Seleucid times. We have but few problem texts from the latter period, but they give us some idea of the type of mathematics familiar to the astronomers of this age. This material is sufficient to assure us that all the essential achievements of Old Babylonian times were still in the possession of the latest representatives of Mesopotamian science. In other words, Babylonian mathematical astronomy was built on foundations independently laid more than a millennium before.

If one wishes to characterize Babylonian mathematics by one term, one could call it "algebra." Even where the foundation is apparently geometric, the essence is strongly algebraic, as can be seen from the fact that frequently operations occur which do not admit of a geometric interpretation, as addition of areas and lengths, or multiplication of areas. The predominant problem consists in the determination of unknown quantities subject to given conditions. Thus we find prepared precisely the tools which were later to become of the greatest importance for astronomy.

Of course, the term "algebra" does not completely cover Babylonian mathemat-

[44] These texts were published in Neugebauer *MKT* (1935–38) and in Neugebauer-Sachs *MCT* (1945). A large part of the *MKT* material was republished in Thureau-Dangin *TMB* (1939). For a general survey see Neugebauer *Vorl.*

ics. Not only were a certain number of geometrical relations well known but, more important for our problem, the basic properties of elementary sequences (e.g., arithmetic and geometric progressions) were developed.[45] The numerical calculations are carried out everywhere with the greatest facility and skill.

We possess a great number of texts from all periods which contain lists of reciprocals, square and cubic roots, multiplication tables, etc., but these tables rarely go beyond two sexagesimal places (i.e., beyond 3600). A reverse influence of astronomy on mathematics can be seen in the fact that tables needed for especially extensive numerical computations come from the Seleucid period; tables of reciprocals are preserved with seven places (corresponding to eleven decimal places) for the entry and up to seventeen places (corresponding to twenty-nine decimal places) for the result. It is clear that numerical computations of such dimensions are needed only in astronomical problems.

The superiority of Babylonian numerical methods has left traces still visible in modern times. The division of the circle into 360 degrees and the division of the hour into 60 minutes and 3600 seconds reflect the unbroken use of the sexagesimal system in their computations by medieval and ancient astronomers. But though the base 60 is the most conspicuous feature of the Babylonian number system, this was by no means essential for its success. The great number of divisors of 60 is certainly very useful in practice, but the real advantage of its use in the mathematical and astronomical texts lies in the place-value notation,[46] which is consistently employed in all scientific computations. This gave the Babylonian number system the same advantage over all other ancient systems as our modern place-value notation holds over the Roman numerals. The importance of this invention can well be compared with that of the alphabet. Just as the alphabet eliminates the concept of writing as an art to be acquired only after long years of training, so a place-value notation eliminates mere computation as a complex art in itself. A comparison with Egypt or with the Middle Ages illustrates this very clearly. Operation with fractions, for example, constituted a problem in itself for medieval computers; in place-value notation, no such problem exists,[47] thus eliminating one of the most serious obstacles for the further development of mathematical technique.

The analogy between alphabet and place-value notation can be carried still further. Neither one was the sudden invention made by a single person but the final outcome of various historical processes. We are able to trace Mesopotamian number-writing far back into the earliest stages of civilization, thanks to the enormous amount of economic documents preserved from all periods. It can be shown how a notation analogous to the Egyptian or Roman system was gradually replaced by a notation which developed naturally in the monetary system and which tended toward a place-value notation. The value 60 of the base appears to be the outcome of the arrangement of the monetary

[45] Incidentally, we also have an example (Neugebauer-Sachs *MCT*, Problem-Text A) of purely number theoretical type from Old Babylonian times (so-called "Pythagorean numbers"); but it should be added that we do not find the slightest trace of number mysticism anywhere in these texts.

[46] Place-value notation consists in the use of a very limited number of symbols whose magnitude is determined by position. Thus 51 does not mean 5 plus 1 (as it would with Roman or Egyptian numerals), but 5 times 10 plus 1. Analogously in the sexagesimal system, five followed by one (we transcribe 5,1) means 5 times 60 plus 1 (i.e., 301).

[47] Example: to add or to multiply 1.5 and 1.2 requires exactly the same operations as the addition or multiplication of 15 and 12.

units.[48] Outside of mathematical texts, the place-value notation was always overlapped by various other notations, and toward the end of Mesopotamian civilization a modified system became predominant. It seems very possible, however, that the idea of place-value writing was never completely lost and found its way through astronomical tradition into early Hindu astronomy.[49] whence our present number system originated during the first half of the first millennium A.D.

10. We now turn to the periods preceding the final stage of Babylonian astronomy which culminated in the mathematical theory of the moon and the planets described above. It is not possible to give an outline of this earlier development because most of the preliminary work remains to be done. A few special problems, however, which must eventually find their place in a more complete picture, can now be mentioned.

In our discussion of the methods used in the lunar and planetary theories, we had occasion to mention the extensive use of periodically increasing and decreasing sequences of numbers. A simple case of this method appears in earlier times in the problem of describing numerically the changing length of day and night during the year. The crudest form is the assumption of linear variation between two extremal values.[50] Two much more refined schemes are incorporated in the texts of the latest period, but it seems very likely that they are of earlier origin. Closely related are two other problems: the variability of the length of the shadow of the "gnomon"[51] and the measurement of the length of the day by water clocks.[52] The latter problem has caused considerable trouble in the literature on the subject because the texts show the ratio 2:1 for the extremal values during the year. A ratio 2:1 between the longest and the shortest day, instead of the ratio 3:2, which is otherwise used,[53] would correspond to a geographical latitude absolutely impossible for Babylon. The discrepancy disappears, however, if one recalls the fact that the amount of water flowing from a cylindrical vessel is not proportional to the time elapsed but decreases with the sinking level.[54] It is worth mentioning in this connection that the outflow of water from a water clock is already discussed in Old Babylonian mathematical texts.[55] This whole group of texts, however, leads to nothing more than very approximate results. This is seen from the fact that the year is assumed, for the sake of simplicity, to be 360 days long and divided into 12 months of 30 days each.[56] This schematic treatment has its parallel in the schemes which we have met in Egyptian astronomy and which we shall find again in early Greek astronomy; we must once more emphasize that elements from such schemes cannot be used for modern calculations, since this would assume quantitative accuracy where only qualitative results had been intended.

The calendaric interest of these problems is obvious. The same is true of the

48 For details see Neugebauer [11] and Neugebauer *Vorl.*, chap. iii § 4. The theory set forth by Thureau-Dangin *SS* (English version Thureau-Dangin [1]) does not account for the place-value notation, which is the most essential feature of the whole system.

49 Cf. Datta-Singh *HHM* I and Neugebauer [12], pp. 266 ff.

50 E.g., Weissbach *BM*, pp. 50–51.

51 Weidner [1], pp. 198 ff.

52 Weissbach *BM*, pp. 50–51; Weidner [1], pp. 195–96.

53 Schaumberger *Erg.*, p. 377.

54 Neugebauer [19].

55 Thureau-Dangin [2] and Neugebauer *MKT*, I, pp. 173 ff.

56 This schematic year of 360 days, of course, does not indicate that one assumed 360 days as the correct length of the solar year. A lunar calendar makes correct predictions of a future date very difficult. The schematic calendar is in practice therefore very convenient for giving future dates which must, at any rate, be adjusted later.

oldest preserved astronomical documents from Mesopotamia, the so-called "astrolabes."[57] These astrolabes are clay tablets inscribed with a figure of three concentric circles, divided into twelve sections by twelve radii. In each of the thirty-six fields thus obtained we find the name of a constellation and simple numbers whose significance is not yet clear. But it seems evident that the whole text constitutes some kind of schematic celestial map which represents three regions on the sky, each divided into twelve parts, and attributing characteristic numbers to each constellation. These numbers increase and decrease in arithmetic progression and are undoubtedly connected with the corresponding month of the schematic twelve-month calendar. It is clear that we have here some kind of simple astronomical calendar parallel (not in detail, but in purpose) to the "diagonal calendars" in Egypt. In both cases these calendars are of great interest to us as a source for determining the relative positions and the earliest names of various constellations. But here, too, the strongest simplifications are adopted in order to obtain symmetric arrangements, and much remains to be done before we can answer such questions as the origin of the "zodiac."

11. Few statements are more deeply rooted in the public mind or more often repeated than the assertion that the origin of astronomy is to be found in astrology. Not only is historical evidence lacking for this statement but all well-documented facts are in sharp contradiction to it. All the above-mentioned facts from Egypt and Babylonia (and, as we shall presently see, also from Greece) show that calendaric problems directed the first steps of astronomy. Determination of the season, measurement of time, lunar festivals—these are the problems which shaped astronomical development for many centuries; and we have seen that even the last phase of Mesopotamian astronomy, characterized by the mathematical ephemerids, was mainly devoted to problems of the lunar calendar. It is therefore one of the most difficult problems in the history of ancient astronomy to uncover the real roots of astrology and to establish their relation to astronomy. Very little has been done in this direction, mainly because of the prejudice in favor of accepting without question the priority of astrology.

Before going into this problem in greater detail, we must clarify our terminology. The modern reader usually thinks in terms of that concept of astrology which consists in the prediction of the fate of a person determined by the constellation of the planets, the sun, and the moon at the moment of his birth. It is well known, however, that this form of astrology is comparatively late and was preceded by another form of much more general character (frequently called "judicial" astrology in contrast to the "genethlialogical" or "horoscopic" astrology just described). In judicial astrology, celestial phenomena are used to predict the imminent future of the country or its government, particularly the king. From halos of the moon, the approach or invisibility of planets, eclipses, etc., conclusions are drawn as to the invasion of an enemy from the east or west, the condition of the coming harvest, floods and storms, etc.; but we never find anything like the "horoscope" based on the constellation at the moment of birth of an individual. In other words, Mesopotamian "astrology" can be much better compared with weather prediction from phenomena observed in the skies than with astrology in the modern sense of the

[57] This name is rather misleading and is merely due to the circular arrangement. Schott [1], p. 311, introduced the more appropriate name "twelve-times-three." Such texts are published in CT 33, Pls. 11 and 12. Cf. also Weidner *Hdb.*, pp. 62 ff. and Schott [1].

word. Historically, astrology in Mesopotamia is merely one form of predicting future events; as such, it belongs to the enormous field of omen literature which is so familiar to every student of Babylonian civilization.[58]

Indeed, it can hardly be doubted that astrology emerged from the general practice of prognosticating through omens, which was based on the concept that irregularities in nature of any type (e.g., in the appearance of newborn animals or in the structure of the liver or other internal parts of a sheep) are indicative of other disturbances to come. Once the idea of fundamental parallelism between various phenomena in nature and human life is accepted, its use and development can be understood as consistent; established relations between observed irregularities and following events, constantly amplified by new experiences, thus lead to some sort of empirical science, which seems strange to us but was by no means illogical and bare of good sense to the minds of people who had no insight into the physical laws which determined the observed facts.

Though the preceding remarks certainly describe the general situation adequately, the historical details are very much in the dark. One of the main difficulties lies in the character of our sources. We have at our disposal large parts of collections of astrological omens arranged in great "series" comprising hundreds of tablets. But the preserved canonical series come mainly from comparatively late collections (of the Assyrian period) and were thus undoubtedly subject to countless modifications. We must, moreover, probably assume that the collection of astrological omina goes back to the Cassite period (before 1200 B.C.)—a period about which our general information is pretty flimsy. From the Old Babylonian period only one isolated text is preserved[59] which contains omina familiar from the later astrology. Predictions derived from observations of Venus made during the reign of Ammiṣaduqa (*ca.* 1600 B.C.) are preserved only in copies written almost a thousand years later[60] and clearly subjected to several changes during this long time. We are thus again left in the dark as to the actual date of the composition of these documents except for the fact that it seems fairly safe to say that no astrological ideas appear before the end of the Old Babylonian period. Needless to say, there are no astrological documents of Sumerian origin.

The period of the ever increasing importance of astrology (always, of course, of the above-mentioned type of "judicial" astrology) is that beginning with the Late Assyrian empire. The "reports" mentioned previously, preserved in the archives of the Assyrian kings, are our witnesses. But here, again, a completely unsolved problem must be mentioned: we do not know how the "horoscopic" astrology of the Hellenistic period originated from the totally different omen type of astrology of the preceding millennium. It is, indeed, an entirely unexpected turn to make the constellation of the planets at a single moment responsible for the whole future of an individual, instead of observing the ever shifting phenomena on the sky and thus establishing short-term consequences for the country in general (even if represented in the person of the king). It seems to me by no means self-evident that this radical shift of the character of astrology actually originated in Babylonia. We shall see in the next section that the horoscopic practice flourished especially in Egypt. It might therefore very well be that the new tendency originated in Hellenistic times

[58] A comprehensive study of the development of the astrological omina literature by E. F. Weidner is in course of publication (Weidner [2]).

[59] Šileiko [1].

[60] Langdon *VT*.

outside Mesopotamia and was reintroduced there in its modified form. It might be significant that only seven horoscopes are preserved from Mesopotamia, all of which were written in the Seleucid period,[61] a ridiculously small number as compared with the enormous amount of textual material dealing with the older "judicial" astrology. It must be admitted, however, that the oldest horoscopes known are of Babylonian origin. On the other hand, at no specific place can all the elements be found which are characteristic for astrology from Hellenistic times onward. Neither Babylonian astrology nor Egyptian cosmology furnishes the base for the fundamental assumption of horoscopic astrology, namely, that the position of the planets in the zodiac decides the future. And, finally, it must be emphasized that the problem of determining the date and place of origin of horoscopic astrology is intimately related to the problem of the date and origin of mathematical astronomy. Horoscopes could not be cast before the existence of methods to determine the position of the celestial bodies for a period of at least a few decades. Even complete lists of observations would not be satisfactory because the positions of the planets in the zodiac are required regardless of their visibility at the specific hour. This shows how closely interwoven are the history of astrology and the history of planetary theories.

IV. THE HELLENISTIC PERIOD

12. Before beginning the discussion of the Hellenistic period, we must briefly describe the preceding development in Greece. Our direct sources of information about astronomy and mathematics before Alexander are extremely meagre. The dominating influence of Euclid's *Elements* succeeded in destroying almost all references to pre-Euclidean writings, and essentially the same effect was produced by Ptolemy's works. Original documents are, of course, not preserved—one must not forget that even our oldest manuscripts of Greek mathematical and astronomical literature were written many centuries after the originals.[62] It is therefore not surprising that our present-day knowledge of early Greek science is much more incomplete and subject to conjecture than the history of Mesopotamian or even Egyptian achievements where original documents are at our disposal. One point, however, can be established beyond any doubt: early Greek astronomy shows very strong parallelism with the early phases of Egyptian and Babylonian astronomy, with respect to scope as well as primitiveness. The astronomical writings of Autolycus[63] and Euclid[64] struggle in a very crude way with the problem of the rising and setting of stars, making very strong simplifications which were forced upon them by the lack of adequate methods in spherical geometry. The final goal is again to establish relations between the celestial phenomena and the seasons of the years; the problem is thus of essentially calendaric interest. In addition to these simple treatises, however, we do find one work of outstanding character: the planetary theory of Eudoxos, Plato's famous contemporary. He made an attempt to explain the peculiarities of a planetary movement known as retrogra-

[61] Two are published by Kugler *SSB*, II, 554 ff., and refer to the years 258 and 142 B.C., respectively. One (probably 233 B.C.) is published in Thompson *AB* 251. Among four unpublished horoscopes, discovered by Dr. A. Sachs, two are very small fragments, one can be dated 235 B.C., and the last was cast for the year 263 B.C.; the last is the oldest horoscope in the world.

[62] The oldest preserved manuscript of Euclid's *Elements* was written about twelve hundred years after Euclid (cf., e.g., Heath *Euclid*, I, p. 47).

[63] Autolycus, ed. Hultsch (Leipzig, 1885).

[64] *Euclidis opera omnia*, Vol. VIII, ed. Menge (Leipzig, 1916).

dation by the assumption of the superposition of the rotation of two concentric spheres around inclined axes and in opposite directions. In this way he reached a satisfactory explanation of the general type of planetary movement and thereby inaugurated a new period in the history of astronomy which was marked by attempts to explain the movements of the planetary system by mechanical models. It contains the nucleus for all planetary theories of the following two thousand years, namely, the assumption that irregularities in the apparent orbits can be explained as the result of superposed circular movements. It is only since Galileo and Newton that we know that the circular orbits do not play an exceptional role and that the great successes of the Greek theory were merely due to the accidental distribution of masses in our planetary system. It is, nevertheless, of great historical interest to see how a plausible initial hypothesis can for many centuries determine the line of attack on a problem, simultaneously barring all other possibilities. Such possibilities were actually contained in the approach developed by the Babylonian astronomers in the idea of superposing linear or quadratic periodic functions. These arithmetical methods were, however, almost completely abandoned by the Greek astronomers (at least so far as we know) and survived only in the treatment of certain smaller problems.

One of these smaller problems is again related to calendaric questions but also to a basic problem of mathematical geography: the determination of the geographical latitude by means of the ratio of the longest to the shortest day. We have already mentioned the Babylonian methods of describing the change in the length of the days by means of simple sequences. These "linear" methods reappear in Greek literature and can be followed far into the early Middle Ages[65] in spite of the invention of much more accurate methods.[66] The term "linear" does not refer so much to the fact that the sequences in question form arithmetic progressions of the first order but is intended to emphasize the contrast with the "trigonometric" method applied to the same problem and explained in the first book of the *Almagest*. Here the exact solution of the problem by the use of spherical trigonometry is given. In contrast thereto, the linear methods yield only approximate results, but with an accuracy which was certainly sufficient in practice, especially when one takes into account the inaccuracy of the ancient instruments used in measuring time. Historically, however, the main interest lies much less in the perfection of the results than in the method employed and in its influence on the further development. A close investigation of early Greek astronomy and mathematics[67] reveals an interesting fact. The determination of the time for the rising and setting of given arcs of the ecliptic, which lies at the heart of the question of the changing length of day and night, appears to be the most decisive problem in the development of spherical geometry. It is typical for the whole situation that a Greek "mathematical" work, the *Sphaerics* of Theodosius (*ca.* 200 B.C.), does not contain a single astronomical remark. The structure and contents of the main theorems, however, are determined by the astronomical problem in question; the methods applied constitute a very interesting link between the Babylonian linear methods and the final trigonometrical methods.

Trigonometry undoubtedly has a very

[65] Neugebauer [13] and [18].

[66] *Almagest* II, 7 and 8. Cf. also *Tetrabiblos* I, 20 (ed. Robbins, p. 94), 21 (ed. Boll-Boer, pp. 46, 47 ff.).

[67] This investigation has been carried out by Olaf Schmidt (doctoral thesis, Brown Univ., 1943 [unpublished]).

long history. We find the basic relations between the chord and diameter of a circle already in use in Old Babylonian texts which employ the so-called "Thales" and "Pythagorean" theorems.[68] In sharp contrast to the Greek models for the movement of the celestial bodies, which operate with circles and therefore necessarily require trigonometrical functions, we find no applications of trigonometry in the cuneiform astronomical texts of the Seleucid period which are exclusively based on arithmetical methods described above.

So far as we know, spherical trigonometry appears for the first time in the *Sphaeric* of Menelaos[69] (*ca.* A.D. 100). The astronomical background of this work is much more outspoken than in Theodosius, but here, too, much is left to the reader, who must be familiar with the methods of ancient astronomy to understand all the astronomical implications. The modern scholar faces an additional difficulty, namely, the modification of the Greek text by the Arabic editors. The Greek original is lost, and what we possess is only the Arabic version made almost a thousand years later. In this interval falls the gradual transformation of Greek trigonometry, operating with chords, to the modern treatment, which uses the sine function. It is well known that this change goes back to Hindu astronomy, where the chords subtended by an angle were replaced by the length of the half-chord of the half-angle,[70] i.e., our "sin α." It is, however, a much more involved question to separate these new methods from those used originally by Menelaos; this question must be answered if we wish to understand the development of ancient spherical astronomy. This, in turn, is necessary in order to appreciate the contributions made by the Hindu-Arabic astronomers which eventually led to the modern form of spherical trigonometry.

13. It is of great interest to see that the very same problem—the determination of rising times—leads to still other methods which are now known partly as "nomography," partly as "descriptive geometry." We have a small treatise, written by Ptolemy, called the *Analemma*.[71] He first introduces in a very systematic way three different sets of spherical coordinates, each of which determines the position of a point on the celestial sphere. Then these coordinates are projected on different planes, and these planes are turned into the plane of construction, just as we do today in descriptive geometry. Finally, certain scales are used to find graphically the relations between different coordinates, again following principles which we now use in nomography. The Arabs used and developed these methods in connection with the construction of sundials.[72] Another method of projection, today called "stereographic," is given in Ptolemy's *Planisphaerium*. The theory of perspective drawing in the Renaissance is directly connected with this work.[73]

The practical importance of the determination of the rising times or the length of the days is not restricted to the theory of sundials. The length of the longest day increases with the geographical latitude, thus giving us the means to determine the latitude of a place from the ratio of the

[68] Cf. Neugebauer-Struve [1], pp. 90–91; Neugebauer *MKT*, I, p. 180; and Neugebauer-Sachs *MCT*, Problem-Text A.

[69] Krause *Men.*

[70] Cf., e.g., Braunmühl *GT*, chap. 3.

[71] Ptolemy, *Opera* II, pp. 187–223. No complete translation of this badly preserved text has yet been published, but an excellent commentary has been given by Luckey [1]. These methods, using descriptive geometry, are of an older date, as is evident from the fact that they are already mentioned by Vitruvius (beginning of our era). Cf. Neugebauer [14] and Luckey [2].

[72] Cf., e.g., Garbers *ES* and Luckey [2].

[73] Ptolemy, *Opera* II, pp. 225–59, translated in Drecker [1]; cf. also Loria in M. Cantor, *Geschichte der Mathematik*, IV, p. 582.

shortest to the longest day. The ratio 3 : 2 accepted by Babylonian astronomers for the ratio of the longest to shortest daylight led the Greek geographers to determine erroneously the latitude of Babylon as 35° (instead of $32\frac{1}{2}$°). This error seriously affected the shape of the eastern part of the ancient map of the world.[74] The precise relationship can only be established by using spherical trigonometry, but here, too, the "linear" methods were applied to various values of the basic ratio in order to give the law for the changing length of the days for the corresponding latitude. It must be remarked, however, that at this stage of affairs the concept "latitude" does not yet actually appear, but the ratio of the longest to the shortest day itself was used to characterize the location of a place. Zones of the same ratio were considered as belonging to the same "clima," a concept which plays a great role in ancient and medieval geography. The difference in character and behavior of nations living in different climates furnished one of the main arguments for the influence of astronomical phenomena on human life.[75]

The second geographical coordinate—the longitude—caused more trouble. The difference in longitude between two places on the earth is essentially equivalent to the difference in local time. But there existed no clocks or signals to compare the local time at far-distant places. Only one phenomenon could be used as a time signal, namely, records of simultaneous observations of a lunar eclipse from two different places. If each observer took note of the local time at which he observed the beginning and end of a lunar eclipse, a comparison of these records would then furnish the needed information. Hipparchus proposed the use of this method for an exact construction of the map of the world, but his program was never carried out. Only one pair of simultaneous observations seems to have been made, the eclipse of 331 B.C., September 20, recorded three hours earlier in Carthage than at Arbela.[76] Actually the difference in local time between these two localities is much smaller, and consequently the ancient map of the world suffers from a serious distortion in the direction from east to west. Here we see one of the most essential differences between ancient and modern science at work. Ancient science suffered most severely from the lack of scientific organization which is so familiar in our own times. In antiquity, generations passed before a new scientific idea found a follower able to use and develop methods handed down from a predecessor. The splendid isolation of the great scholars of antiquity can only be paralleled with the first beginnings of the new development in the European Renaissance. It seems to me beyond any doubt that even centers like Alexandria or Pergamon during their height would appear very poorly equipped if compared with a modern university of moderate size. And these centers themselves were few and practically isolated at any particular time; and at all times they were dependent upon the mood of some autocratic ruler. No wonder that the great achievements of antiquity are either the result of priestly castes of sufficiently stable tradition or of a few ingenious men who expended tremendous energy in restoring and enlarging the structure of a science known to them from the written legacy of their predecessors. One must not think

[74] For the determination of the size of the earth by Eratosthenes (about 250 B.C.), Marinus of Tyre (about A.D. 100), and Ptolemy (about A.D. 150), see Mžik *EGM*, pp. 96 ff., and, in general, Heidel *GM*, chap. xi. Cf. also Honigmann *SK* and Neugebauer [13].

[75] E.g., *Tetrabiblos* II, 2.

[76] Ptolemy *Geographia* i. 4. 2 (ed. Nobbe, p. 11). Cf. also Mžik-Hopfner *PDE*, p. 21, n. 3. For Hipparchus' program see Strabo *Geography* i. C. 7; also Berger *GFH*, pp. 12 ff.

that mathematics and astronomy, like the popular philosophical systems or the art of rhetoric, were taught in the same manner from generation to generation. Three centuries separate Hipparchus from Ptolemy, one Eudoxos from Euclid, Euclid from Archimedes and Apollonius. To be sure, the literary tradition was never interrupted between these outstanding men, but most of the intermediate literature at best merely preserved and commented. This explains not only why ingenious ideas were frequently lost (e.g., Archimedes' methods of integration) but also why it was so easy to destroy ancient science almost completely in a very short time. Astronomy alone had a slight advantage because of its practical usefulness in navigation, geography, and time-reckoning, supplemented by the fortunate accident that the Easter festival followed the lunar calendar of the Near East, thus sanctioning lunar theory when other secular sciences fell into total desuetude.

The extreme paucity of scientists at almost any given time in antiquity gave rise to another phenomenon in Greek literature: the publication of commentaries and popularizing works. A work like the *Almagest*, written in purely scientific style, was certainly unintelligible to the majority of people who needed or wanted to know a modest amount of astronomy. Hence books were written which attempted to explain Ptolemy's text sentence by sentence,[77] or which gave abstracts accompanied by explanations of the main principles as far as this could be done without mathematics.[78] We can observe the same phenomenon in geography. The first chapter of Ptolemy's *Geography*[79] contains a very interesting theory of map projection, whereas the remaining twelve chapters constitute an enormous catalogue of localities from all over the then known world and the corresponding values of longitude and latitude to be plotted into the network which was to be constructed according to the method explained in the first chapter. This, again, was not geography for the entertainment of the general reader. To satisfy popular tastes, there was another literature, represented by works like Strabo's *Geography*.[80] These more pleasant writings furnished serious competition to the strictly scientific literature and determined to a large extent the character of the field in late antiquity and the Middle Ages.

14. For the modern historian of ancient astronomy it is therefore of the greatest value to have an additional source of astronomical literature in which the earlier tradition was kept alive without interruption for a much longer period: the astrological texts. We have already mentioned that astrology in the modern use of the word appeared very late in antiquity. The art of casting horoscopes can be said to be a typical Hellenistic product, the result of the close contact between Greek and oriental cultures.[81] We possess Greek papyri from Egypt from the beginning of our era to the Arabian conquest showing us the application of astronomical methods in a great number of specific horoscopes and in minor astronomical treatises.[82] In addition, an enormous astrological literature is preserved, catalogued during the last fifty years in the twelve volumes of the *Catalogus* by Cumont and

[77] The commentaries of Pappus and Theon of Alexandria (and presumably of Hypathia) are of this type. For these texts cf. Rome *CPT*.

[78] Represented, e.g., by Theon of Smyrna (second cent. A.D.) or Proclus (fifth cent. A.D.).

[79] Edited by Nobbe (1843). The first chapter is excellently discussed by Mžik and Hopfner *PDE*.

[80] Edited and translated in the "Loeb Classical Library" by H. L. Jones (8 vols.; 1917–32).

[81] Cf., e.g., Capelle [1], who shows that only weak traces of astrological ideas in Greek literature can be followed as far back as 400 B.C.

[82] Concerning horoscopes, see above, n. 17. Examples of astronomical treatises are Pap. Ryl. 27, 464, 522/24, 527/28, or Curtis-Robbins [1].

his collaborators.[83] Finally, Vettius Valens, who wrote shortly before Ptolemy,[84] and Ptolemy himself as the author of the famous *Tetrabiblos*, must be mentioned.[85]

Modern scholars have not yet made full use of this vast material. The reason is only too clear: the amount of work to be done surpasses by far the power of a single individual, and the work itself is certainly not very pleasant. The astronomical part must be extracted from occasional remarks, short computations, and similar instances submerged beneath purely astrological matter of a very unappealing character. But this work must eventually be done and will give valuable results. As an example might be mentioned the question of discovering the principle according to which the equinox was placed in the zodiac. This question must be answered, for on it depend our calculations in the determination of constellations, chronology, etc. Moreover, systematic checking of astrological computations will frequently yield information about the character of the astronomical tables used at the time.

We touch here upon a point of great importance for the modern attitude toward ancient astronomy. The usual treatment of ancient sciences as a homogeneous type of literature is very misleading. It is necessary to realize that very different levels of astronomy or mathematics were coexistent, almost without mutual contact or interference. One misses the essential points in the understanding of ancient astronomy if one naïvely considers various documents in their chronological order. Even works by the same person must sometimes be separated from one another. Ptolemy's *Almagest* is purely mathematical, the *Tetrabiblos* (written after the *Almagest*)[86] is purely astrological, and his *Harmonics*[87] contains a chapter on the harmony of spheres employing concepts of the planetary movements which contains such strong simplification of the actual facts that one would try in vain to find similar assumptions in any of the other works of Ptolemy. In other words, it is necessary to evaluate each text in its proper surrounding and according to its traditional style. One cannot, for example, speak without qualification of the contact between Babylonian and Greek astronomy. Such a contact might even have worked in opposite directions in different fields. For instance, we have already referred to the possibility that Hellenistic astrology returned to Babylonia in the form acquired in Egypt or Syria, whereas observational material from Mesopotamia undoubtedly influenced Greek mathematical astronomy deeply. In general, it can be said that the growth of ancient sciences shows much more irregularity and stratification than modern scientists, accustomed to the fact of the uniform spread of modern ideas and methods, are prone to assume.

The lack of uniformity in the whole field of ancient astronomy in general necessarily interferes also with the investigation of any special problem. We have already mentioned the fact that astrology in the Assyrian age differed considerably from the horoscopic type which prevailed in late antiquity and the Middle Ages. But there exists a third type, standing between the omina type ("when this and this happens in the skies, then such and such a major event will be the consequence") and the individual birth horoscope, namely, the "general prognostication," explained in full detail in the first two books of the *Tetrabiblos*. This type of

83 *CCAG*. Cf. also Boll [2].

84 Kroll *VV*.

85 Ptolemy, *Opera* III, 1, and "Loeb Classical Library" (ed. F. E. Robbins).

86 This follows from the introduction to the *Tetrabiblos*.

87 Düring *HP* and *PPM*.

astrology is actually primitive cosmic physics built on a vast generalization of the evident influence of the position of the sun in the zodiac on the weather on earth. The influence of the moon is considered as of almost equal importance, and from this point of departure an intricate system of characterization of the parts of the zodiac, the nature of the planets, and their mutual relations is developed.[88] This whole astronomical meteorology is, to be sure, based on utterly naïve analogies and generalizations, but it is certainly no more naïve and plays no more with words than the most admired philosophical systems of antiquity. It would be of great interest for the understanding of ancient physics and science in general to know where and when this system was developed. The question arises whether this is a Greek invention, replacing the Babylonian omen literature, which must at any rate have lost most of its interest with the end of independent Mesopotamian rule, whether it precedes the invention of the horoscopic art for individuals or merely represents an attempt to rationalize the latter on more general principles.[89] Thus we see that even in a single field of ancient astronomical thought the most heterogeneous influences are at work; the analysis of these influences has repercussions on almost every aspect of the study of ancient civilizations.[90]

15. The same branching-off into very different lines of thought must also be recognized in the development of Greek mathematics. The line of development characterized by the names of Eudoxus, Euclid, Archimedes, and Apollonius is to be separated sharply from writings like Heron[91] and Diophantus[92] or the *Arithmetic* of Nicomachus of Gerasa.[93] Here, again, the question of oriental influence cannot be discussed as one common phenomenon. Egyptian calculation technique and mensuration were certainly continued in similar works in Hellenistic Egypt and found their way into Roman and medieval practices. At the same time, Babylonian numerical methods influenced Alexandrian astronomy. How Babylonian algebraic concepts eventually reached Greek writers like Diophantus is still completely unknown, but that it did is supported by the strong parallelism in methods and problems.[94] Equally lacking is detailed information as to the revival of these methods in Moslem literature.[95] On the other hand, the problems which emerged from the discovery of the irrational numbers are undoubtedly of Greek origin. It is, however, not correct to consider writings of the same person as equally representative of "Greek" mathematics. Those parts of Euclid's *Elements* (the majority of the work) which deal more or less directly with the problem of irrational numbers are, as we said before, Greek. Most likely of equally Greek origin is Euclid's astronomical treatise called *Phenomena*,[96] which is written on so elementary a level that nobody would attribute it to the author of the *Elements* if the authorship were not so firmly established. And, finally, Euclid's *Data*[97] contains the treatment of purely algebraical problems by geometrical means—which can be interpreted as the direct geometrical transla-

[88] For the whole complex of the ancient justifications of astrology, see Duhem, *SM*, II, 274 ff.

[89] This is the assumption of Kroll [1], p. 216, for the tendency exhibited in Ptolemy's *Tetrabiblos*.

[90] Cf. the excellent survey of this situation in Boll [2].

[91] First century A.D.; cf. for this date Neugebauer [14], pp. 21 ff.

[92] Usually dated about A.D. 300; cf., however, Klein [1], p. 133, n. 23.

[93] Greek text ed. Hoche (Leipzig, 1866); English translation: D'Ooge-Robbins-Karpinski *Nic.*

[94] Vogel [2]; Gandz [3].

[95] Gandz [1], [2], [3].

[96] *Opera* VIII; cf. above, p. 16.

[97] *Opera* VI.

tion of methods well known to Babylonian mathematics.[98] These methods of "geometrical algebra" in turn determine the whole structure of Apollonius' theory of conic sections.[99]

Greek mathematics is by far the best-investigated field of ancient science (and of the history of science in general);[100] the situation with respect to the source material is very good[101]—except where only Arabic manuscripts are preserved.[102] But one must not forget that also this tradition suffers from severe gaps. This is due not only to the destruction of manuscripts over a period of two thousand years but also to the effect of literary influence. I refer not only to the above-mentioned elimination of older treatises by the overshadowing of the great works of the Hellenistic period. The Greeks themselves contributed to the distortion of the picture of the actual development by inventing seemingly plausible stories where the real records were already lost. The oft-repeated stories about Thales, Pythagoras, and other heroes are the result.[103] We should now realize that we know next to nothing about earlier Greek mathematics and astronomy in general and about the contact with the Near East and its influence in particular. The method which involves the use of a few obscure citations[104] from late authors for the restoration of the history of science during the course of centuries seems to me doomed to failure. This amounts to little more than an attempt to understand the history of modern science from a few corrupt quotations from Kant, Goethe, Shakespeare, and Dante.

16. Undoubtedly the most spectacular advances in the history of astronomy until very recent times were scored in the theory of the planets. The catch-words "Ptolemaic" and "Copernican" refer to different assumptions as to the mechanism of the planetary movement. This is not the place to underline the fact that the Copernican theory is by no means so different from or so superior to the Ptolemaic theory as is customarily asserted in anniversary celebrations,[105] but we must briefly analyze Ptolemy's own claims to having been the first one who was able to give a consistent planetary theory.[106] This claim seems to contradict not only the existence of pre-Ptolemaic planetary tables in Roman Egypt as well as in Mesopotamia but also Ptolemy's own reference to such texts. What Ptolemy means, however, becomes clear if one reads the details of the introduction to his own theory. He requires an explanation of the planetary movement by means of a combination of uniform circular movements which refrains from simplifications like the assumption of an invariable amount for the retrograde arc and similar deviations from the actual observations. Indeed, in order to remain in close agreement with the observations, Ptolemy had to overcome difficulties which Hipparchus was not able to

[98] Neugebauer [15].

[99] Zeuthen *KA* and Neugebauer [16].

[100] Best exposition: Heath *GM* and *MGM* and Euclid. A selection of texts is given in Thomas *GMW*.

[101] Most of the texts are edited in the Teubneriana collection.

[102] Menelaos alone is now edited (Krause *Men.*), but Books v, vi, and vii of Apollonius' *Conic Sections* are still unavailable in a modern edition. Archimedes' construction of the heptagon is published in a free translation of the Arabic version in Schoy *TLAB*, pp. 74–91; cf. also Tropfke [1].

[103] As an example might be mentioned the criticism of the story of the Thales eclipse by Pannekoek [3], p. 955; Dreyer *HPS*, p. 12, n. 2; Neugebauer [9], pp. 295–96. Cf. also Frank, Plato, or Heidel [1].

[104] The fragments collected by Diels *VS* not only give an extremely incomplete picture of the lost writings but were certainly very much distorted by the authors from whose works they are taken. One needs only to look at the picture of oriental writings obtained from Greek tradition as compared with the originals.

[105] The correct estimate can be found in Thorndike *HM*, Vol. V, chap. xviii.

[106] *Almagest* IX, 2.

master and which led Ptolemy to a model which is very close to Kepler's final solution of the problem, by assuming not only an eccentric position of the earth but also an eccentric point around which the movement of the planetary eccenter appears to be uniform. The resulting orbit is of almost elliptical shape with these two points as foci.[107] This whole theory is closely related in method to the explanation of the "evection" of the moon (a periodic perturbation of the moon's orbit discovered by Ptolemy) by a combination of eccentric and epicyclic movements. Both theories are real masterpieces of ancient mathematical astronomy which far surpassed all previous results.

It is not surprising that Ptolemy's results overshadowed all previous works. All that we know about his forerunners comes mainly from the *Almagest* itself. We hear that Hipparchus used eccenters and epicycles for the explanation of the anomalies in the movement of the sun and the moon,[108] and we learn about theorems for such movements proved by Apollonius.[109] This brings us to the very period (about 200 B.C.) from which the oldest cuneiform planetary texts are preserved—computed, however, on entirely different principles. These cuneiform texts cover the two centuries down to the time of Caesar. A direct continuation, chronologically speaking, but of still another type, are planetary tables from Egypt, written in Demotic or Greek.[110] These tables give the dates at which the planets enter or leave the signs of the zodiac. Such tables were known to Cicero[111] and are most likely the "eternal tables" quoted with contempt by Ptolemy.[112] We do not know how these tables were computed, and their occurrence in Greek as well as in Demotic leaves us in doubt as to their origin—showing us only the degree of interrelation we can expect in Hellenistic times.

The most interesting question would, of course, be to learn more about Hipparchus' astronomy. He is most famous as the discoverer of the precession of the equinoxes. Though this fact cannot be doubted,[113] underlining its importance lays the wrong emphasis on a phenomenon which gained its importance only from Newton's theory, which showed that precession depends on the shape of the earth and thus opened the way to test the theory of general gravitation by direct measurements on the earth. For ancient astronomy, however, precession played a very small role, requiring nothing more than sufficiently remote and sufficiently reliable records of observations of positions of fixed stars. The change in positions must then eventually become evident; and little difficulty was encountered in incorporating this slow movement into the adopted model of celestial mechanics. What we actually need to appreciate in Hipparchus' contribution must be derived from a careful study of all relevant sections of the *Almagest*, not by the schematic method of obtaining "fragments" from direct quotations but by a comparison of Ptolemy's methods and the older procedures which he frequently mentions. That such an approach can lead to well-defined results has recently been shown in the theory of eclipses.[114]

17. One of the most important prob-

[107] Cf. Schumacher [1] for the Ptolemaic theory of Venus and Mercury. For the Greek planetary theory in general, see Herz *GB* I.

[108] *Almagest* III, 4.

[109] *Almagest* XII, 1 (=Apollonius, ed. Heiberg, II, 137).

[110] Neugebauer [3]. Cf. above, p. 5.

[111] Cicero *De divinatione* ii. 6, 17; cf. also ii. 71. 146.

[112] *Almagest* IX, 2.

[113] Schnabel's attempts (Schnabel [1]) to prove that precession was taken into consideration in the cuneiform texts are, to say the least, inconclusive and in part based on mere scribal errors.

[114] Schmidt [1].

lems in connection with Hipparchus is, of course, the problem of the dependence of Hipparchus (and Greek astronomy in general) on Babylonian results and methods. Whatever the conclusions derived from a deeper knowledge of Hipparchus' astronomy may turn out to be, one thing is clear: the century between Alexander's conquest of the Near East and Hipparchus' time is the critical period for the origin of Babylonian mathematical astronomy as well as for its contact with Greek astronomy. Since Kugler's discoveries, which showed the exact coincidence between numerical relations in cuneiform tablets and in Hipparchus' theory,[115] no one has doubted Babylonian priority. It is an undeniable fact that the Babylonian theory is based on mathematical methods known already in Old Babylonian times and does not show any trace of methods considered to be characteristically Greek. The problem remains, however, to answer the question: What caused the sudden outburst of scientific astronomy in Mesopotamia after many centuries of a tradition of another sort? On what background can we understand, for example, the report[116] that the "Chaldaean" Seleucus from Seleucia on the Tigris[117] completed the heliocentric theory, previously proposed as a hypothesis by Hipparchus? Greek influence on late Babylonian astronomy must not be denied or asserted on aprioristic grounds, if we really want to understand a phenomenon of great historical significance.

These remarks are not intended to make Greek influence alone responsible for the new developments in Mesopotamia. As a matter of fact, this answer would only raise the equally unsolved question why Greek astronomy suddenly emerged from many centuries of primitiveness to a scientific system. The alternative, Greek or Babylonian, might even exclude the right answer from the very beginning. It also seems possible that the rise of mathematical astronomy in Hellenistic times resulted from the suddenly intensified contact between several types of civilization, in some respects to be paralleled with the origin of modern science in the Renaissance. In other words, neither the Greeks nor the Orientals might have been alone responsible for the new development but rather the enormous widening of the horizon of all members of the culture of the Hellenistic age. One result of this process was probably the new attitude toward the relationship between the individual and the cosmos, expressed in the new form of horoscopic astrology. In this case it is quite evident that Egypt and Greece—and perhaps Syria as well—contributed about equally much to the refinement and spread of this new creed. It is equally possible that the contact between Greek scholars, trained to think in geometrical terms which Greek mathematics had developed in the fifth century, and Babylonian astronomers, equipped with superior numerical methods and observational records, brought into simultaneous existence two closely related types of mathematical astronomy: the treatment by arithmetical means in Babylonia and the model based on circular movements in the Greek centers of learning in the eastern Mediterranean. It may well be that competition, not borrowing, was the chief contributor to the initial impetus.[118] At any rate, it is clear that each detail in the development of Hellenistic astronomy which we will be able to understand better will reveal a new aspect in the fascinating process of the

[115] Kugler *BMR*, p. 40.

[116] Plutarch *Plat. quaest.* vii. 1. 1006 C (ed. Bernardakis, *Moralia*, VI, 138). Cf. also Heath, *AS*, pp. 305 ff. and Duhem *SM*, I, 423 ff.

[117] Strabo xvi. 739. Seleucus may have lived about 150 B.C.

[118] Neugebauer [17], pp. 30–31.

creation of the new world which was destined to become the foundation of the Roman and medieval civilizations.

The unique role of the Hellenistic period in the field of sciences, as in other fields, can be described as the destruction of a cultural tradition which dominated the Near East and the Mediterranean countries for many centuries, but also the founding of a new tradition which held following generations in its spell. The history of astronomy in the Hellenistic age is especially well suited to demonstrate that the great energies liberated by the disintegration of an old cultural tradition are very soon transformed into stabilizing forces of a new tradition, which includes about as many elements of development as of stagnation.

V. SPECIAL PROBLEMS

18. Every research program in a complex field will face the need of constant modification and adjustment to unforeseen complications and new ramifications. Problems can arise and results be obtained without having been anticipated in the original question. The context of a mathematical text, for example, can determine with absolute certainty the meaning of a word otherwise only vaguely defined; sign-forms in a papyrus which is exactly dated by astronomical means may furnish valuable information for purely paleographical problems. From dates and positions given in Demotic astronomical texts, it follows that the Alexandrian calendar introduced by Augustus was used by Egyptian scribes only a few years after the reform,[119] very much in contrast to the common opinion that the Egyptians were especially conservative in general and in calendaric matters in particular. In short, from few, but solidly established, facts we can learn more than from all general speculations.

One of the problems which at first sight lies very much outside the history of ancient astronomy is the study of social and economic conditions of the ancient civilizations. There are, however, several points of contact between these studies and astronomy. We are indebted to Cumont for a masterly investigation of the information contained in the astrological literature from Hellenistic Egypt.[120] His results are not only of interest for the history of ancient civilization but also illustrate very well the background of the men who used and transmitted the astronomical material known to us from the planetary tables or from Vettius Valens. It turns out that the soil in which these practices were rooted was essentially Egyptian, in spite of the use of the Greek language in the documents. This is in perfect harmony with the close parallelism between Greek and Demotic planetary texts mentioned above and shows the constant interaction of Greek and native influences in Hellenistic Egypt. It also shows how dangerous it is to decide the authorship of Hellenistic doctrines or methods simply on the basis of such superficial grounds as the language used.

The analogous question for Babylonia seems to be easier to answer. The Mesopotamian origin of the astrological omina cannot be doubted. We would, however, like to know more about the background of the astronomers of the latest period. It is well known that the names of three Babylonian astronomers appear in Greek literature[121] and that two of them actually were found on astronomical tablets, though in an unclear context. For one particular place, the famous city of Uruk in South Babylonia, we can go much further. It can be shown that the scribes and owners of our texts belong to one of two

[119] Neugebauer [6], p. 119.

[120] Cumont *EA*. See also Kroll [1].

[121] Cumont [1].

"families," or perhaps "guilds," of scribes who frequently call themselves scribes of the omen-series "Enuma Anu Enlil."[122] We can follow the work of these scribes very closely for almost a hundred years until the school of Uruk ceased to exist, probably because of the Parthian invasion of Babylonia in 141 B.C. In contrast thereto, the school of Babylon survived the collapse of the Greek regime, as is proved by a continuous series of astronomical texts down to 30 B.C. This is an interesting result in comparison with the assumption that Babylon practically ceased to exist after the Parthian occupation. The grouping of our texts according to well-defined schools is also of interest from another point of view. It can be shown that two different systems of computation existed side by side for a long time. Competing schools of this sort constitute a phenomenon which is usually considered characteristic for Greek culture.

19. Countless thousands of business documents are preserved from all periods of Mesopotamian history. For the urgently needed investigation of ancient economics, a precise knowledge of the metrological systems is of the greatest importance. Unfortunately, the scientific study of Babylonian measures has been sadly neglected. Fantastic ideas about the level and importance of astronomy in the earliest periods of Babylonian history led to theories which brought measures of time and space in close relationship with alleged astronomical discoveries. We know today that all these assumptions of the early days of Assyriology must be abandoned and that Babylonian metrology must be studied from economic and related texts clearly separated according to period and region. For the determination of Old Babylonian relations between various measures, the mathematical texts are of great value because they contain numerous examples which give detailed solutions of problems in which metrological relations play a major role. The consequences of such relations, established with absolute certainty, are manifold. For example, we now know from Old Babylonian mathematical texts the measurements of several types of bricks[123] as well as the peculiar notation used in counting bricks. It is evident that such information is of importance for the understanding of contemporary economic texts dealing with the delivery of bricks for buildings, thus leading to purely archeological questions. Metrological relations are also needed if we wish to gain an insight into wages and prices.[124] Returning to our subject, it must be said that metrology is of great importance not only for the history of the economics of Mesopotamia but also for purely astronomical problems. Distances on the celestial sphere are measured in astronomical texts by units borrowed from terrestrial metrology. The comparison between ancient observation and modern computations thus requires a knowledge of the ancient relations between the various units. This problem is by no means simple because our astronomical material belongs to relatively late periods, Assyrian and Neo-Babylonian, and the metrological system of these times is much more involved than the Old Babylonian. Mathematical texts would certainly be of great help here too, but the few tablets from this period are so badly preserved that they present us with at least as many new questions as they answer. Neo-Babylonian economic texts will therefore furnish the main point of departure for the study

[122] For this series cf. Boll-Bezold-Gundel *SS*, pp. 2 ff., and Weidner [2].

[123] Neugebauer-Sachs *MCT*, Problem-Text O and Sachs [1].

[124] Waschow [1], p. 277, found, in discussing mathematical texts, that the value of the area-measure "še" must be changed by a factor 60 against older assumptions. It is obvious how such facts influence the interpretation of economic texts.

of the latest phase of Mesopotamian metrology and its astronomical applications.

It might be mentioned, in this connection, that theories about direct relationship between early Mesopotamian metrology and astronomy also gave rise to the rather unfortunate concept of high accuracy in the determination of weights, measures of length, etc. It is of great importance to realize that the absolute values of all metrological units are subject to great margins of inaccuracy and local and temporal variations. The first step in a historical investigation of Mesopotamian metrology must therefore be to establish from economic and mathematical texts the *ratios* between the units; these ratios have an incomparably better chance of showing unformity than the absolute values deduced from accidental archeological finds.

20. Closely related to metrological problems is the question of the accurate identification of ancient star configurations. Much work remains to be done before it will be possible to give a reliable history of the topography of the celestial sphere in general, or even of the zodiacal constellations.[125] In spite of attempts to make Egypt responsible for many forms,[126] the predominant influence of Babylonian concepts on the grouping of stars into pictures must be maintained. But neither Babylonian nor Egyptian developments are known in detail. The identification of Egyptian constellations is especially difficult, mainly because it must be based on relations between the times of rising and setting and therefore depends on elements which are grossly schematized in the texts at our disposal. The situation in Mesopotamia is slightly better because we have actual observations in addition to the schematic lists, at least for the later periods which are of special importance for the Hellenistic forms of the constellations.

For the period following the publication of the *Almagest*, we must take into account the possibility of still other complications. We know from explicit remarks in the *Almagest* that Ptolemy's star catalogue introduced deviations from older catalogues.[127] Astrological works, however, may very well have maintained pre-Ptolemy standards both with respect to the boundaries of constellation and the counting of angles in the zodiac. We have already mentioned the stubborn adherence of astrological writers to methods of computation which were made obsolete by the development of spherical trigonometry.[128] For the modern historian it is therefore of importance to establish the specific standard according to which a given document was written, especially when chronological problems are involved.

21. While metrology is a much-needed implement for economic history and the understanding of ancient astronomy, astronomy itself serves general history in chronological problems. Chronology is the necessary skeleton of history and owes its most important fixed points to astronomical facts. We need not emphasize the use of reports of eclipses, especially solar eclipses, for the determination of accurate dates to form the framework into which the results of relative chronology must be fitted. It must be underlined, however, that the available material is by no means exhausted. A better understanding and reinvestigation of the reports of the Assyrian astronomers will certainly furnish new information of chronological value. It must be stated, on the other

[125] The best summary is given by the Boll-Gundel article, "Sternbilder," in Roscher *GRM*, Vol. VI (1937), cols. 867–1072.

[126] Cf. esp. Gundel *DD* and *HT* and the criticism of Schott [2].

[127] *Almagest* VII, 4 (ed. Heiberg, p. 37).

[128] Cf., e.g., *Tetrabiblos* I, 20 (ed. Robbins, pp. 94–95).

hand, that not too much is to be expected from older material. In order to make ancient observations accessible to modern computation, a certain degree of accuracy must be granted; this accuracy seems to be missing in the earlier phases of the development of astronomy. This, for instance, makes the older Egyptian material so ill suited for chronological purposes. For later periods, however, Egypt has furnished and will furnish much information from astrological documents. It is particularly calendaric questions, such as the use of eras and similar problems, which have been illuminated by the dating of horoscopes.

The great variety of calendaric systems, local eras, and older methods of dating raises many difficulties in ancient chronology. This difficulty was clearly felt also by ancient astronomers and was the cause of the early use of consistent eras in Babylonian and Greek astronomy. The Babylonian texts always use the Seleucid Era, whereas Ptolemy reduces all dates to the Nabonassar Era but uses the Old Egyptian years of constant length. This crossing of Egyptian and Babylonian influences is paralleled by the subdivision of the day into hours. The Egyptians divided the day into twelve parts from sunrise to sunset, thus obtaining hours whose length depended on the season. The Babylonian astronomers used six subdivisions of day and night, but these units were of constant length. Combining the Egyptian division into 24 hours with the Babylonian constancy of length, the Hellenistic astronomers used "equinoctial" hours for their computations and solved the problem of finding the relationship between seasonal and equinoctial hours by spherical trigonometry.[129] One sees here again what a multitude of relations, problems, and methods contributed to shape concepts such as a continuous era or the 24-hour day which are so familiar to us today.

Ancient chronology and the accurate analysis of ancient reports have turned out to be of interest even to a modern astronomical problem. In 1693 Halley discovered the fact[130] that the moon's position appeared to be advanced compared with the expected position as computed from positions recorded by Ptolemy. This "acceleration" can be explained by a slow increase in the length of the solar day or by a decrease in the rotational velocity of the earth. Such a decrease is caused by tidal forces,[131] and it is of great interest to determine the amount as accurately as possible. For this purpose, accurate positions of the moon in remote times are of great value, and such positions can, indeed, be derived from records in cuneiform texts.[132] Modern measurements of high precision can thus be supplemented by observations in antiquity.

22. Not only are Hellenistic astronomy and Hellenistic astrology the determining factors for the astronomy and astrology of the Middle Ages in Europe, but its influence is equally important for the development of astronomical methods and concepts in the Middle and Far East. We must therefore at least mention an enormous field which still awaits systematic research: Hindu science. This does not mean that there is not an extensive literature on this subject; indeed, even a small number of original texts are published.[133] The main trouble lies, however, in the tendency of the majority of publications by Hindu authors to claim priority for Hindu discoveries and to deny foreign in-

[129] *Almagest* II. 9,

[130] Edm. Halley, "Emendationes ac notae Abatênii observationes astronomicas, cum restitutione tabularum lunisolarum ejusdem authoris," *Philosophical Transactions*, **17** (1693), No. 204, pp. 913–21.

[131] Cf., e.g., Jeffreys [1].

[132] P. V. Neugebauer [1].

[133] For the literature until 1899, see Thibaut *AAM*. The best discussion of Hindu astronomy is still Burgess *SS* (1860).

fluence, as well as in the opposite tendency of some European scholars. This tendency has been especially strong so far as Hindu mathematics is concerned,[134] and it is aggravated by the inadequate publication of the original documents, from which usually only scattered fragments are cited in order to prove some specific statement. As a result, there is no means today to obtain an independent judgment from the study of the original texts which are preserved in enormous number, though of relatively late date for the most part.

The situation with respect to Hindu astronomy is not much better. There can be little doubt that the original impetus came from Hellenistic astronomy; the use of the eccentric-epicyclic model alone would be sufficient proof even if we did not also find direct witness in the use of Greek terminology.[135] This fact is interesting in itself, but it may very well be that the period of reception lies between Hipparchus and Ptolemy; systematic study might therefore reveal information about pre-Ptolemaic Greek astronomy no longer preserved in available Greek sources. Hindu astronomy would in this case constitute one of the most important missing links between late Babylonian astronomy and the fully developed stage of Greek astronomy represented by the *Almagest.*

The fundamental difficulty in the study of Hindu astronomy lies in the character of the preserved textual material. The published and commented texts consist exclusively of cryptically formulated verses giving the rules for computing certain phenomena, making it extremely difficult to understand the actual process to be followed. It is evident, on the other hand, that no astronomy of an advanced level can exist without actually computed ephemerids. It must therefore be the first task of the historian of Hindu astronomy to look for texts which contain actual computations. Such texts are, indeed, preserved in great number, though actually written in very late periods. Poleman's catalogue[136] of Sanskrit manuscripts in American collections lists about a hundred such manuscripts in the D. E. Smith collection in Columbia University in New York. In their general arrangement, these texts are reminiscent of the cuneiform ephemerids from Seleucid times and must reveal many details of the Hindu theory of the planetary movement if attacked by the same methods which have proved so successful in the case of the Babylonian material. The complete publication of this material is an urgent desideratum in the exploration of oriental astronomy.

As mentioned above, the texts in the D. E. Smith collection are of very recent origin, only a few centuries old. This does not mean that the methods used are not of very much earlier date. This is shown by the investigation of one of these texts,[137] which deals with the problem of the varying length of the days during the year. Though written about 1500, the computations are based on methods going back to a much older period. Analogous results can be expected in the remaining material, and there is no reason to assume that the D. E. Smith collection exhausts all the preserved material.

23. In the preceding sections we have frequently touched on methodological questions. In closing, I wish to underline a few principles in a more general way. As is only natural, the study of the development of ancient science began under the

134 Cf., e.g., Datta-Singh *HHM* (reviewed in Neugebauer [12]).

135 Thibaut *AAM*, pp. 43 ff. The Babylonian ratio 3 : 2 for the ratio between the longest and shortest days of the year also occurs in India (Thibaut *AAM*, pp. 26–27; Kugler *BMR*, pp. 82 and 195), though it would be suitable only for the latitude of the northern corner of India. For the planetary theory, see Kugler *BB*, p. 120; Schnabel [2], p. 112; Schnabel [1], p. 60.

136 Poleman *CIM*, pp. 231 ff. See also Emeneau *PIT*, pp. 318 ff.

137 Schmidt [2].

influence of the ancient tradition. Herodotus, Diodorus, the commentators of Plato, etc., were the sources which determined the picture of the early stages of Greek and oriental mathematics and astronomy. But while students of political history, art, economics, and law learned in the early days of systematic archeological research to consider this literary tradition about the ancient Orient as nothing more than a supplementary source to be checked by the original documents, the majority of historians of the exact sciences have remained in a stage of naïve innocence, repeating without criticism the nursery stories of ancient popular writers. This is all the more surprising because many of these stories should have revealed their purely fictitious character from the very beginning. Every invention considered of basic importance is attributed to a definite person or nation: Thales "discovered" that a diameter divides the area of a circle into two equal parts, Anaximandes and several others are credited with the discovery of the obliquity of the ecliptic, the Egyptians discovered geometry, the Phoenicians arithmetic—and so on, according to an obvious pattern of naïve restoration of facts the origins of which had been totally forgotten. Modern authors then add stories of their own, such as the idea that the construction of the pyramids required mathematics, the assumption of supposedly marvelous skies of Mesopotamia,[138] and the notion of Egyptian Stone Age astronomers industriously determining the heliacal rising of Sirius or carrying out a geodetic survey of the Nile Valley.

It is clear that the replacement of the traditional stories by statements based exclusively on results obtainable from the original sources will not be very appealing. This is the inevitable result in the development of every science; for increased knowledge means giving up simple pictures. In the history of science, an additional element must be added to the steady increase of complexity resulting from a better understanding of our sources. Not only do we learn to interpret our material more accurately but we also learn to see everywhere the immense gaps in our preserved sources. We will more and more be forced to admit that many, and essential, steps in the development of science are hopelessly destroyed; that we, at best, are able to sketch mere outlines of the history of science during certain sharply limited periods; and that many of the driving forces might actually have been quite different from those which we customarily restore on the analogy of later periods.

One consequence of this situation seems to me to be evident: unless the history of science now enters the stage of specialization, it will lose all value in the framework of historical research. It must be clearly understood that the history of science must work with methods and must consider its problems from viewpoints which correspond to the methods and standards of other branches of historical research. The idea must definitely be abandoned that the history of science must adapt its level to the alleged requirements of the teaching of the modern fields of science. The intrinsic value of this research must be seen in its contribution to our understanding of the historical processes which shaped human civilization, and it must be made clear that such an understanding cannot be reached without the closest contact with the other historical fields. The call for specialization is not very popular. I am convinced, however, that a well-founded insight into the details of a single essential step in the development is at present of higher value and more fascinating than any attempt at general syn-

[138] For the poor conditions of actual observation cf. Koldewey *WB*, p. 192; Vogt [1], pp. 38–39; cf. also Boll [1], pp. 48 and 157.

thesis. It is ridiculous to believe that we are anywhere able to reach "final" results in the study of the development of human civilization. But the overwhelming richness of all phases of human history can be appreciated only if we occupy ourselves with the real facts as accurately as possible and do not attempt to hide their manifold aspects under the veil of hazy generalizations or let our judgment be guided by the naïve idea of human "progress." Every synthesis written fifty years ago is now completely antiquated and at best enjoyable for its literary style; the careful study of the original works of the ancients, however, will reveal to everyone and at any time the development of their achievements.[139]

The call for specialization must not be misunderstood as a plea for the disregard of the general outlines of the historical conditions. On the contrary, specialized work can be accomplished successfully only if the points of attack are selected under constant consideration of possible interference from other problems and other fields. It is indeed the most gratifying result of detailed research on a well-defined problem that it necessarily uncovers relationships which are of primary importance for the understanding of larger historical processes. The actual working program, however, needs restriction and minute detail work. The most essential task is that of making the original sources accessible as easily as possible in their best available form. By the indefatigable work of Heiberg, Hultsch, Tannery, and many others, we possess today a great part of the extant writings of the Greek scientists in excellent editions. We owe to Sir Thomas Little Heath many brilliant commentaries and translations of Greek mathematicians.[140] To make Greek and oriental source material more generally accessible, supplemented, of course, by modern translations and commentaries, will be the foremost problem of the future. The extension of this program to include medieval material, on the one hand, and Middle Eastern documents, on the other, appears as a logical consequence, worthy of the serious efforts of all scholars who wish to contribute to the understanding of the past of our own culture.

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[139] An excellent example is Delambre *HAA*, published in 1817 and still not surpassed or even equaled because of its direct contact with the original sources.

[140] On the other hand, much remains to be done to repair the harm caused by classical philologists who made their editions inaccessible to modern scientists by translating them into Latin instead of a modern language. Great opportunities have been spoiled by this absurd attitude. It has fortunately never occurred to Orientalists to translate their texts into Hebrew. It should be mentioned, however, that the Arabic version of Euclid's *Elements* was published in Latin(!) translation by Besthorn, Heiberg, and others (Copenhagen, 1897–1932).

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AJSL	*American Journal of Semitic Languages and Literatures.*
Almagest	*See* Ptolemy.
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